THE MATHEMATICAL
ASSOCIATION OF VICTORIA

MATHS TREATS

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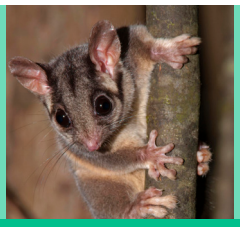
THE POSSUM



USING COMPUTERS FOR MATHEMATICAL PROOFS

A mathematics conjecture is a claim that has not yet been proven or disproven. A mathematics proof is an argument based on deductive reasoning that demonstrates a logical link between stated definitions and a mathematical claim. Mathematical proofs include direct proof, proof by mathematical induction, proof by contradiction (like our chess problem), and proof by exhaustion (where every possible case is individually checked). The first computer-assisted proof was the four-colour theorem in 1976. Historically, computer-assisted proofs have been large-scale proofs where it has been proven, for example, that if a conjecture is true up to a very large (finite) integer, it will be true for all integers. A computer can be used to check this, and so complete the proof. Computers have proved, by exhaustion, that the largest known prime number is the Mersenne prime $2^{82\,589\,933} - 1$. Finally, artificial intelligence and machine learning are promising new tools for the development of mathematical proofs.

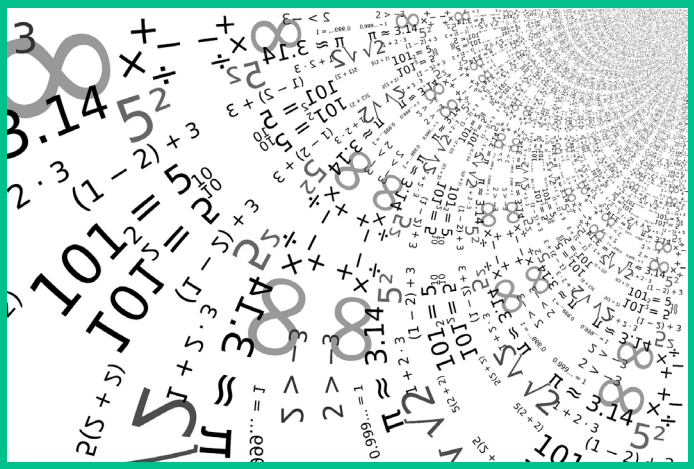
MAKING CONJECTURES

Dynamic geometry software makes it easy to construct shapes and explore lines, angles, and intersections. Such explorations can lead to conjectures about the properties of shapes, and stronger definitions of the essential elements of the shapes. These are not proofs, but may lead to proofs.

ACTIVITY

Use dynamic geometry software (e.g., GeoGebra) to develop some conjectures. For example, investigate the properties of perpendicular bisectors of the sides of a triangle, angle bisectors of a triangle, and altitudes of a triangle. What can you say about the diagonals of a square, rectangle, rhombus, or parallelogram? Explore the properties of the shape formed by the midpoints of the sides of a quadrilateral. What can you say about the angles inscribed within a semicircle or a circle? (Marrades & Gutiérrez, 2000).

TESTING CONJECTURES



Spreadsheets enable a large number of calculations to be performed efficiently. Using a spreadsheet to build an argument requires a systematic approach to ensure that many cases are identified and tested. To be considered a proof, all cases must be tested and verified, or one counterexample must be identified.

ACTIVITY

Use a spreadsheet to build a convincing argument that the following statements are true or false: The sum of two even integers equals an even integer. The square of an odd number is also odd. The sum of the first n odd, positive integers is n^2. Can an irrational number raised to an irrational power give rise to a rational number? Do you notice any patterns in your results? Can you prove, or disprove, these statements by other means (e.g., visual or algebraic methods)? What other conjectures can you make and test?

REFERENCES AND FURTHER READING

Search Wikipedia (and other sources) for conjecture, mathematical proof, computer-assisted proof, direct proof, mathematical induction, proof by contradiction and proof by exhaustion.

Luciana the Possum. (2019). *Vinculum*, *56*(2), 24.

Marrades, R., & Gutiérrez, A. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics*, *44*(1/2), 87-125. https://doi.org/10.1023/A:1012785106627

Images: Leadbeater possum - Steve Kuiter, others from Pixabay.